ANTICRACKS: A NEW THEORY OF FRACTURE INITIATION AND FRACTURE PROPAGATION IN SNOW

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ABSTRACT: Shear failure models have traditionally dominated the research on slab avalanche initiation in snow science, but evidence has accumulated that they tell us only part of the story. In these models weak snowpack layers are regarded as compounds of zero thickness, but in reality they often consist of ice grain aggregates of finite thickness and high porosity. When such granular aggregates fail, the remaining fragments can rearrange in a tighter packing and the weak layer undergoes a reduction in volume. This compaction has fundamental consequences for understanding slab avalanche release. We present a new theory of fracture initiation and propagation in weak layers, based on the mathematical concept of mixed-mode anticracks. Energy criteria for crack propagation and skier triggering are obtained from the theory. Initiation and propagation of anticrack-mediated fracture are illustrated on field measurements.

KEYWORDS: fracture, anticrack, packing order.

INTRODUCTION

Seasonal snow is a cohesive granular form of ice that occupies a large volume per unit mass (Blackford, 2007). Its internal structure is a filigrane matrix of ice grains that may suddenly crumble or crush into a more compact form when pressure is applied. The consequences of this compaction for the theory of fracture in snow have not yet been explored. We examine in this article how fracture can undercut a cohesive slab when the ice grains in the weak layer pack tighter during the fracture process.

FRACTURE MODES

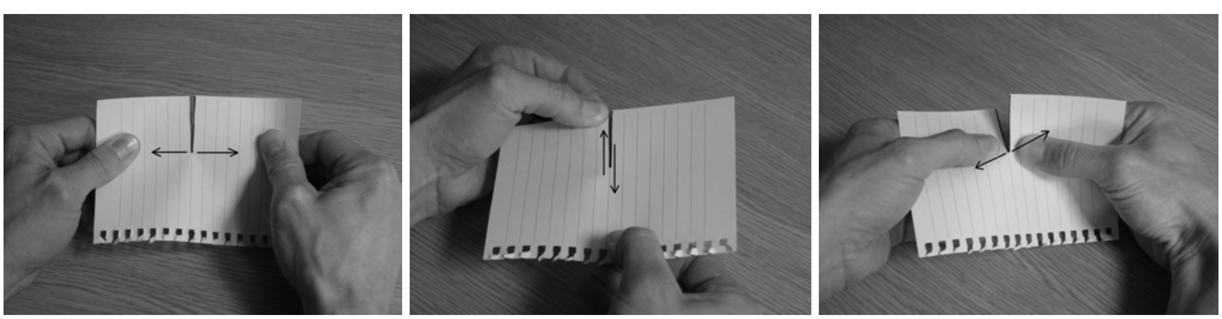
In fracture mechanics we distinguish three basic fracture modes (Fig.1). Mode I is often called the opening mode, because a space is opened between the crack faces under tensile loading. Modes II and mode III are called shear modes, because the crack faces move in parallel direction along each other (slipping over each other). At first glance, mode I is not relevant during weak layer failure, as there are no forces which could lift the slab to create an open space. The weight of the snow, as well as loads acting on the surface such as skiers, compress the snow and certainly do not lift it. Thus it seems natural to conclude that mode I fracture is not relevant and that the shear modes II and III should be responsible for fracture initiation and propagation along weak layers. Models considering shear stability and shear fracture have indeed been the main approach to slab avalanche initiation since the time when the classical shear band model for clay was applied for the first time to slip surfaces in snow some 30 years ago (McClung, 1979). The shear fracture concept is known not only to snow scientists but also to interested backcountry skiers and has entered various textbooks on snow avalanches. At the same time field workers, mountain guides, snow scientists, teachers of avalanche courses, and experienced skiers are well aware of phenomena such as sudden subsidence of the snowpack, whumpfs propagating across flat terrain, failed avalanches (Birkeland et al., 2006), remotely triggered avalanches (Johnson, 2000), and know well that whumpfs are an unmistakable warning sign of snowpack instability. However, none of these phenomena can be explained by shear fracture and shear stability models. In fact, as we shall see, all these phenomena involve some amount weak layer collapse (Johnson, 2004), but a consistent physical theory for any of these observations was not available up to the present.

LIMITATIONS OF THE SHEAR MODEL

In the shear model of slab avalanche formation one assumes weak layers to have zero thickness or, more generally, constant density during fracture. Consequently there is no subsidence of the snowpack. This facilitates very much the calculations needed to determine when fracture propagation becomes energetically self-sustained. The central equations of the shear model are two conditions for spontaneous, self-sustained crack growth. One equation applies to cracks that are short in comparison with the slab thickness and is attributed to Louchet et al. (2002), the other applies to long cracks in comparison with slab thickness and has been proposed by McClung (1979). However both these expressions lead to some contradictions with facts: (i) According to the shear crack theory, the critical size at which a crack starts to propagate rapidly should increase strongly as the slope angle decreases. But recent field experiments done by Gauthier and Jamieson (2008) show instead that the critical size of artificially induced cracks slightly increases or remains about constant as the slope angle increases. This surprising result is a serious problem as it questions one of the core equations of the shear model. (ii) Skiers moving on flat or weakly inclined terrain may trigger remote avalanches on adjacent slopes. However, according to the shear crack model, cracks can neither initiate nor propagate on nearly flat ground as there is no driving force: The mathematical structure of the models simply does not allow for this possibility.

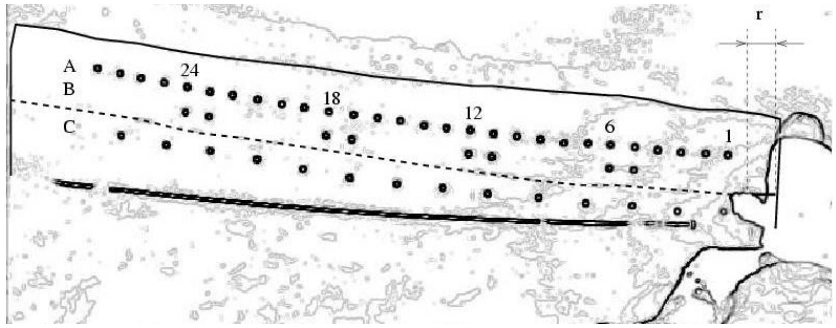
MIXED-MODE ANTICRACK MODEL

These problems can be resolved by noting that incompressibility during fracture is an unlikely assumption in a porous, anisotropic and brittle material such as snow, where weak layer collapse is frequently observed (van Herwijnen et al., 2008). However, to account for this possibility the calculations for snow slope stability need to be entirely redone.

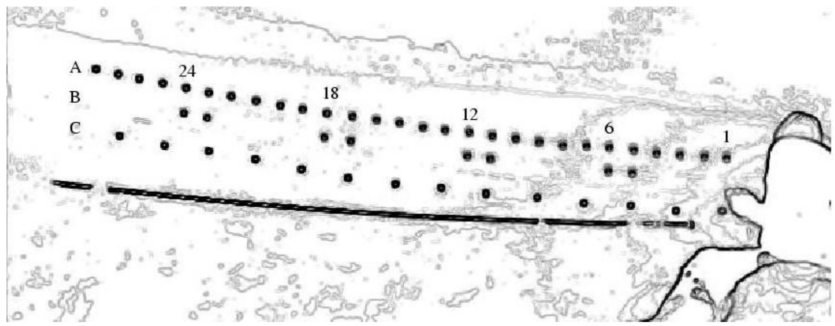


a b c

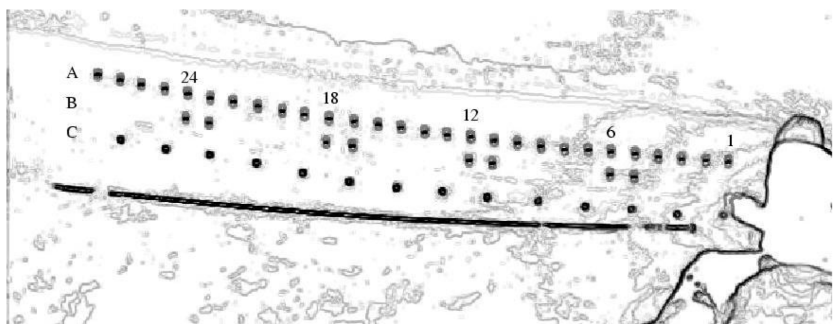
Figure 1: Fracture modes. a: mode I, b: mode II, c: mode III. A fourth mode, known as anti-mode I or simply anticrack is the opposite of a mode I crack, when fracture propagates by pushing the crack faces together (discussed in the text). All modes can occur simultaneously, in which case one speaks of mixed-mode (anti)cracks.



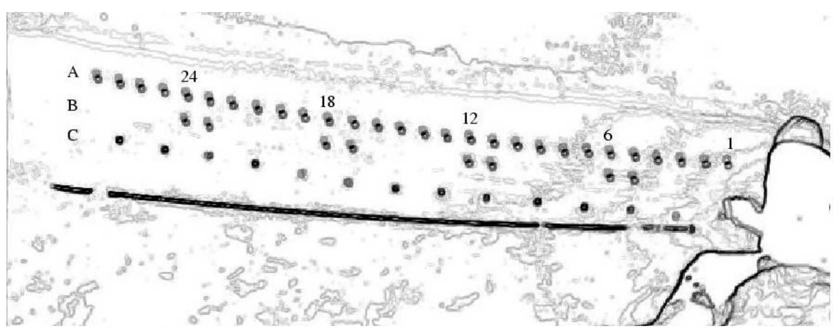
a: t = 5.60 s (ultimate frame before propagation)



b: superposition t = 5.60 s and t = 5.68 s.



c: superposition t = 5.60 s and t = 5.76 s



d: superposition t = 5.60 s and t = 5.84 s

Figure 2: Long-beam PST-type experiment with multiple markers. The contour plots are extracted from video frames and superposed, showing the displacements of the markers during fracture. a: reference frame, dashed line: weak layer, full line: contour of snow sample, *r* denotes the critical length. b: markers 1-20 moving vertically; c:markers 1-28 moving vertically; d: final slip and stop after fracture crossed the full sample length. Davos Stilli, 01/2008; weak layer: surface hoar. For details see van Herwijnen et al. (2008)

Deformation-induced compaction of granular assemblies is a phenomenon of everyday life. Think of a box of coffee that needs to be refilled. After you pour in the grains, there may sometimes be just a bit of leftover that does not fit into the box. You give the box a little shake, the grains arrange in a tighter order, and free space is created. Now, replace the loosely packed coffee grains with facets, depth hoar or surface hoar crystals, place a dry cohesive slab on top, and you essentially get the problem we consider in this contribution.

The idea that collapse plays an important role in weak layer failure is in fact older than the shear model. Bohren and Beschta (1974) gave a first although sketchy description of the collapse process: "Our guess is that [what is observed] is a progressive collapse of the snow, initiated by foot, due to the existence of structurally weak depth hoar at the bottom of the snowpack. Depth hoar [...] is characterized by weak intergranular bonding, large grains with stepped surfaces, and is believed to be a major cause of slab avalanches." With much insight, Bohren and Beschta remark that the collapse wave that propagates the fracture is neither an elastic shear wave nor an elastic longitudinal wave. Over the past decade, the physics of collapsing snow has regained attention. Experimental work by Johnson et al. (2004), van Herwijnen and Jamieson (2005), van Herwijnen et al. (2008), Gauthier and Jamieson (2008a), Sigrist and Schweizer (2007), and Simenhois and Birkeland (2006) produced a number of data and observations that point to the necessity of a new theory of avalanche initiation. These contributions were of key importance for the development and testing of new theories of fracture initiation and propagation in snow slopes. A first attempt to model weak layer collapse in terms of the propagation of linear flexural waves was undertaken by Johnson (2000). However, as collapse is an intrinsically non-linear and irreversible process, it is problematic to model it in terms of the propagation of a linear wave (Heierli, 2005, 2006). Nevertheless we would like to point out the importance of Johnson's research as a turning point in avalanche science.

Weak layers and potential fracture planes in snow consist of loose and highly porous aggregates of ice grains. When such granular aggregates fail, the fragments can rearrange in more tightly packed order and the weak layer undergoes a reduction in volume. This implies the formation of a cavity under a section of unsupported slab.

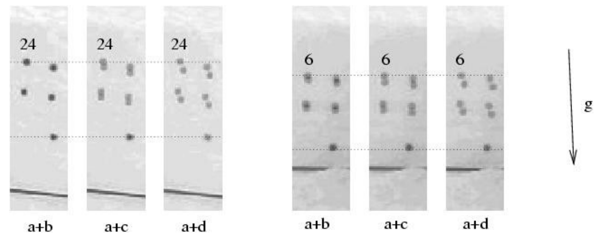


Figure 3: Details of the superposition of video frames a to d (see Fig.2) at markers 6 and 24. Marker 6, close to the saw cut, starts moving before marker 24, away to the notch. Displacement is vertical first, then slope-parallel, and finally stops. The substrate below the weak layer does not move. For quantitative results, see van Herwijnen et al. (2008).

We previously discussed why the 'opening' fracture mode I is not available in snow. However, in the fracture mechanical and geophysical literature this mode has a counterpart known as anti-mode I or simply anticrack (Knight an Knight 1972, Fletcher and Pollard 1981, Green et al. 1990, Sternlof et al. 2005). The prefix 'anti-' refers to the fact that the deformation field around the fractured area is equal and opposite to the deformation field around a classical mode I crack, that is, the anticrack has the astonishing ability to expand under compression. In many materials this is not possible - in fact, anticracks can occur only if fracture goes along with a reduction in volume. But that is exactly what happens in snow: the anticrack can be thought of as a cavity which expands when the surrounding material is compressed by the weight of the snow or by a passing skier.

Careful examination of Fig. 2 illustrates the process. At the front of the anticrack, where the weak layer crumbles, the slab loses its support and starts falling. The released potential energy provides the energy needed to fracture adjacent sections of the weak layer. The fall stops when the tightest possible packing order of the ice grains is attained. The slab starts to slide once the anticrack has cut through the sample (Fig. 2d, Fig. 3). In a recent publication, Heierli, Gumbsch and Zaiser (2008) calculated the energy of formation of anticracks in weak layers and derived the consequences the initiation of slab avalanches. It was found that mixed-mode anticracks can propagate over large areas and delaminate the snowpack under loads that are much less than those needed for propagating a pure shear crack. The outcome of the calculations agrees very well with field experience and field experiments (Birkeland et al. 2006, Sigrist and Schweizer 2007, Gauthier and Jamieson, 2008). For the development of the anticrack model we refer to Heierli, Gumbsch and Zaiser (2008). In the following we present the main results of the new model.

RESULTS

According to the anticrack model, slab avalanche release must be understood as a two-stage process. The calculations show that, in a first stage, nucleation and propagation of a mixed-mode anticrack delaminate the slab from the underlying snow (or a solid bed surface) over a wide area. In a second stage, the crack faces get again into contact and frictional forces between the slab and the bed surface then decide whether the slab will slide, causing an avalanche, or subside, causing a whumpf. Two separate conditions for fracture propagation and for overcoming frictional forces emerge from the calculation. The condition for fracture propagation is weakly dependent on slope angle when anticracking is important, and especially allows for propagation over horizontal terrain. The condition for overcoming frictional forces is sharply thresholded by the slope angle of the bed surface. The two conditions become identical in the shear model, which can therefore be considered as a one-stage process.

The two-stage process has the consequence that avalanches can be triggered remotely from less inclined terrain. The anticrack can propagate across stretches where the snow can only subside, and subsequently climb or descend into adjacent slopes, in which the slab is able to slide after the passage of the crack front, resulting in an avalanche.

Because the deformation in the vicinity of the anticrack releases comparatively large amounts of potential energy, fracture can be more easily initiated than previously thought in terms of the shear model. Typically critical sizes for anticrack propagation may be small, between a few centimeters and a meter at most. Thus very small cracks in the weak layer may be the cause of very large avalanches. Experimentally, this is confirmed by cut lengths of 10 cm to 40 cm which are sufficient in PST-type experiments to induce self-sustained fracture propagation (Gauthier and Jamieson, 2008a).

Once an anticrack of critical size is formed, the energy associated with crack growth is favorable for its sudden expansion in all directions. Unlike the energy of formation of a shear crack, which depends strongly on slope angle, the energy of formation of a mixed-mode anticrack is independent of slope angle for small angles and only weakly depends on slope angle for larger angles.

The theory may give additional hints on what snow properties to test in the field to assess the risk of triggering an avalanche. An essential factor is the ability of the network of ice granules to pack tighter when the bonds between the granules break, as in this case large amounts of potential energy are convertible to fracture energy. In other words, in the field one would need to test for the ability of the material to occupy less volume after fracture than before fracture. However, there are limitations. The theory shows that the amount of subsidence of the slab required to propagate the anticrack can be small. Often, a collapse height as small as 1 mm already provides a 10-fold of the energy required for fracture propagation. Such small collapse heights can be difficult to detect. They can make the crack look like a shear crack from afar, but energetically and this is what matters - it is by no means a shear crack.

The fracture energy of a weak layer can be quantitatively assessed by carrying out PST experiment (Gauthier and Jamieson, 2008) and applying the equations we specifically provided for this experimental set-up (Heierli, Gumbsch and Zaiser, 2008). This can be programmed on a reasonable pocket calculator.

The model also leads to a physical understanding of skier triggering. We have shown in Heierli and Zaiser (2008) that the load of a skier can have the effect of reducing the energy barrier for fracture propagation to nil, even in absence of a pre-cracked zone in the weak layer. In this case, crack propagation and delamination of slab and bed surface may occur spontaneously.

The calculation presented in Heierli, Gumbsch and Zaiser (2008) is based upon very general assumptions. The calculation contains the shear models of Louchet and of McClung as limiting cases and, additionally, closes the gap between these models for intermediate size cracks. Both Louchet's and McClungs' models can be derived from the mixed-mode anticrack model by assuming incompressible fracture of the weak layer and carrying out some simple calculations.

NEXT STEPS

The next step, on which we are currently working, is to answer the question how an anticrack evolves into a collapse wave. This mainly theoretical work is accompanied by field work presented in a another contribution to this ISSW conference (van Herwijnen et al., 2008). The available experimental results and theoretical calculations agree remarkably well. A propagation velocity of 21 m/s was predicted by the model calculations, and subsequently a velocity of 20 m/s was measured (van Herwijnen et al., 2008). The research on collapse waves may also give essential clues for the arrest of fracture propagation which was recently identified as a key problem for understanding avalanche hazard by Gautier and Jamieson (2008b).

CONCLUSION

The practical work of avalanche experts, field workers and avalanche forecasters implies frequent and difficult decisions, which need to be constantly re-assessed as snow conditions change. It is therefore a legitimate request that the causes for snow avalanches should be identified as clearly as possible by snow science and materials science.

Because of its explanatory value, the present theory could be an important step in understanding what is going on at the very moment when a slab avalanche initiates. The key to the occurrence of anticracks in snow is an arrangement of ice grains that allows for rearranging in a tighter packing order during fracture.

The theory resolves the previously problematic explanation of whumpfs, remotely triggered avalanches, triggering in absence of pre-cracked zones, the correlation of whumpf occurrence and snowpack instability and gives a deeper understanding of spontaneous cracking of snow and skier triggering. The compressive forces in the snowpack contribute to -and sometimes dominate- the initiation and propagation of fracture in weak layers. The calculation leads to a natural understanding of recent field experiments. The theory is based on a clear mathematical formulation, and a particular solution has been worked out to apply to PST-type experiments (Gauthier and Jamieson, 2008a).

Field observers can test in situ, how the packing order of grains in a weak layer is affected when the weak layer is disturbed, and how much friction appears between crack faces immediately after fracture. We expect this to reduce the formidable gap between understanding and predicting snow avalanches.

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